PG – 609

I Semester M.Sc.Degree Examination, January 2015
(CBCS)
MATHEMATICS
M 104 T : Ordinary Differential Equations

Time : 3 Hours Max. Marks : 70

Instructions : *i) Answer **any five** questions.*
*ii) All questions carry **equal** marks.*

1. a) Establish Liouville's formula for $L_n y = 0$. Discuss any two consequences of this formula. **9**

   b) Find the Wronskian of the solutions of a homogeneous differential equation of order three which has a root m_1 with multiplicity 3 on an interval I containing zero. **5**

2. a) State Lagrange's identity and verify it for $x^2y'' + 7xy' + 8y = 0$. **8**

   b) Prove that the operator $L = \frac{d^k}{dx^k} P(x) \frac{d^k}{dx^k}$ where P(x) is a real valued function is a self – adjoint operator. **6**

3. a) State and prove sturm's separation theorem on the zeros of the solutions of a self – adjoint differential equation. **7**

   b) Using the method of variation of parameters, find the general solution of

   $$(x^2 + 1)y'' - 2xy' + 2y = 6(x^2 + 1)^2$$

   ($y = x$ is one independent solution of the corresponding homogeneous equation). **7**

4. a) Define sturm – Liouville problem. Find the eigenvalues and eigen functions of

   $y'' + \lambda y = 0;\ y(0) = 0 = y(\pi)$. Also, expand e^x in terms of an orthonormal eigen functions of the above problem. **10**

   b) Show that the eigenvalues of a self – adjoint eigenvalue problem are always real. **4**

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5. a) Obtain the ordinary, regular and irregular singular points (finite), if any, of the Laguerre equation :

$$xy'' + (1-x)y' + \alpha y = 0 .$$ **7**

b) Using Frobenius method obtain the general solution of the hermite equation :

$$y'' - 2xy' + 2\alpha y = 0 .$$ **7**

6. a) Prove the orthogonal property of Chebyshev polynomials. **7**

b) The Laguerre equation :

$xy'' + (1-x)y' + \alpha y = 0$ has a regular singular point at infinity. Prove or disprove this statement. **7**

7. a) Find the fundamental matrix solution of the following system of equations :

$$\frac{dx}{dt} = 6x - 3y + e^{5t} \; ; \frac{dy}{dt} = 2x + y + 4 .$$ **7**

b) Determine the critical points of the system :

$$\frac{dx}{dt} = x + y \quad ; \quad \frac{dy}{dt} = 3x - y$$

Discuss the nature and stability of the critical points and obtain the general solution of the system. **7**

8. a) Determine the nature and stability of the critical points of the nonlinear systems.

$$\frac{dx}{dt} = 1 - y \quad ; \quad \frac{dy}{dt} = x^2 - y^2 .$$ **7**

b) Determine the stability of the critical point (0,0) of the following system using the Liapunov direct method :

$$\frac{dx}{dt} = -x^5 - y^3 \quad ; \frac{dy}{dt} = 3x^3 - 5y^3 .$$ **7**